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Time Series Properties of Canadian Real Interest Rates

by

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ABSTRACT

Recent inquiries -- see Mishkin's (1992) approach to test for the existence of short and long-run Fisher effects and Rose's (1988) examination of the time series properties of real interest rates -- have focused on U.S. data. We expand this investigation to Canada over the post-war period. Standard unit root tests suggest that evidence supporting the Fisher effect is tenuous and that the real interest rate is not stationary. These unit root tests, however, are known to have low power. Furthermore, the underlying data generating process -- for the real interest rate as well as its determinants -- might not be purely autoregressive. Monte Carlo experiments have been performed which allow for a richer ARIMA representation. We employ alternative approaches (see Campbell and Mankiw (1987) and Cochrane (1988)) to address the issue of stationarity, presenting a more accurate characterisation of the time series model generating the real rate. Using a structural VAR model that includes the real interest rate, we examine and quantify (in the spirit of Blanchard and Quah (1989)) the effects of nominal and real disturbances.

RÉSUMÉ

Les récents travaux -- par exemple ceux de Mishkin (1992) sur l'existence d'un effet Fisher à court et à long terme et de Rose (1988) sur les propriétés temporelles des taux d'intérêt réels -- sont basés sur des données américaines. Ce travail élargit le débat à la situation canadienne d'après-guerre. Les tests standards de racine unitaire supportent faiblement l'hypothèse d'un effet Fisher et concluent à la non-stationnarité des taux d'intérêts réels. Les tests standards de racine unitaire sont cependant reconnus pour être peu puissants. De plus, le processus générateur de données sous-jacent -- autant pour le taux d'intérêt réel que pour ses déterminants -- n'est probablement pas purement autorégressif. Dans ce travail, l'utilisation d'expérience à la Monte Carlo permet une bien meilleure représentation ARIMA. Des approches alternatives (celles de Campbell et Mankiw (1987) et de Cochrane (1988)) ont aussi été utilisées pour étudier la question de stationnarité. Ces approches présentent une caractérisation beaucoup plus précise du processus générant les données temporelles du taux d'intérêt réel. À l'aide d'un modèle VAR structurel, ce travail examine (dans l'esprit de Blanchard et Quah (1989)) les effets des perturbations nominales et réelles sur les taux d'intérêt réels.

Section I

I.1 Introduction

Real interest rates have an important influence on the economy, as the recessions of the early 1980s and 1990s demonstrate. They play a critical role in the formulation and transmission of macroeconomic policy through their influence on key economic variables. The level of the real rate of interest in relation to the growth rate of the economy plays an important role in the determination of fiscal policy. Consequently, the determinants and behaviour of real rates are of intrinsic interest to policy makers, who must decide the direction of macroeconomic policy, and forecasters who need to determine where real interest rates may be headed in the future.

The real interest rate is a crucial determinant of investment and savings and is involved in virtually all intertemporal decisions.¹ Naturally, its time series characteristics would be of interest in themselves, however, there are other reasons why we should be interested in the properties of the real interest rate. In some models, stationarity of the real interest rate has important implications for the ability of short-term interest rates to predict future inflation, though it is not clear whether the real rate is indeed stationary. By examining its underlying process, we are able to draw conclusions about the effects of shocks to the real rate. As well, we can distinguish and quantify the effects of different types of shocks to the real interest rate within a structural VAR framework.

This paper's structure is as follows. Section II of this paper examines various econometric approaches used to construct *ex ante* real rates. In Section III we test for non-stationarity in short-term real rates and the existence of the Fisher effect. Section IV approaches the issue of stationarity from different perspectives and presents estimates of the "size" of the random walk and examines the dynamic effects of different types of shocks to the real interest rate. Finally, Section V presents comments and conclusions about various issues raised in our paper and suggests areas for further work.

Section II

II.1 Constructing *Ex Ante* Real Interest Rates

The *ex ante* real interest rate r_t^k (of a k period bond held for k periods) is defined from the Fisher (1930) equation,

$$r_{t,k} = i_{t,k} - \pi_{t,k}^e \quad (2.1)$$

¹Rose, 1988, p. 1095.

where $i_{t,k}$ is the nominal interest rate at time t on a k period bond and $\pi'_{t,k}$ is the expected rate of inflation at time t for the period t to $t+k$. The aspect of equidistant horizons for the expected inflation rate and the term maturity of the bond in the Fisher equation is often overlooked in the literature.

There are a number of approaches which have been used to estimate *ex ante* real rates. This section examines three approaches and applies them to Canadian data. We consider different estimation techniques that are appropriate in terms of the time horizon for real rates. The first approach we consider, employed by Cumby and Mishkin (1986), involves estimating *ex post* real rate regressions and then using the fitted values as estimates of short-term *ex ante* real rates. For these short-term rates we assume that actual and expected inflation differ by an unforecastable error term. Over a short time horizon of three months, we would not expect individuals to systematically over- or under-estimate inflation, on average the difference between their forecasts of inflation and actual inflation would be zero. This assumption allows us to use the fitted values from *ex post* real interest rate regressions as estimates of *ex ante* rates.

The second approach involves using a proxy of expected inflation -- trend inflation -- as estimated by a Hodrick-Prescott filter. The Hodrick-Prescott proxy of expected inflation has been extensively used in OECD country estimates of real rates. In the third approach, expectations about future inflation are based on forecasts from an ARMA model of the inflation process, an approach employed by Barro and Sala-i-Martin (1990). Over a longer time horizon, say of five years, the uncertainty regarding future inflation is much greater. In this case, we assume that individuals know the underlying data generating process for inflation. With this knowledge they can forecast inflation well into the future, however, there will likely be more uncertainty surrounding these estimates. Individuals continually update their inflation forecasts when new information becomes available.

II.2 The Cumby-Mishkin Approach

The *ex ante* real rate in the Fisher equation (2.1) above is unobservable. However, it is possible to calculate the *ex post* real rate ($epr_{t,k}$) using the nominal interest rate and the actual rate of inflation observed over the period t to $t+k$, $\pi_{t,k}$.

$$epr_{t,k} = i_{t,k} - \pi_{t,k} \quad (2.2)$$

If we assume that actual and expected inflation differ by an unforecastable error term ($\varepsilon_{t,k}$), substituting the Fisher equation into the *ex post* real rate equation yields,

$$epr_{t,k} = r_{t,k} - \varepsilon_{t,k} \quad (2.3)$$

Implicitly we are assuming rational expectations, $E(\varepsilon_{i,k}|\phi_t) = 0$, where ϕ_t is all available information at time t . Thus, the forecast error of inflation is unforecastable in the sense that it is not correlated with any information at time t . Also we assume there exists a set of variables X_t that are included in ϕ_t and are correlated with $r_{i,k}$. Cumby and Mishkin (1986) (referred to as CM below) consider the linear projection of $r_{i,k}$ onto X_t , $P(r_{i,k}|X_t) = X_t \beta$ a logical choice for an estimate of $r_{i,k}$. The projection equation is,

$$r_{i,k} = X_t \beta + u_{i,k} \quad (2.4)$$

where $u_{i,k} = r_{i,k} - P(r_{i,k}|X_t)$ and $P(u_{i,k}|X_t) = 0$. The *ex post* real rate can be redefined as,

$$epr_{i,k} = X_t \beta + u_{i,k} - \varepsilon_{i,k} = X_t \beta + \eta_{i,k} \quad (2.5)$$

This equation can be estimated by OLS, where the fitted values from this regression serve as the estimates of the *ex ante* real interest rate, $\hat{r}_{i,k} = X_t \hat{\beta}_{OLS}$.

CM use monthly observations where the holding period or term maturity is three months. This means that the data will be overlapping. Using OLS still yields consistent estimates of β , however, the variance-covariance matrix of $\hat{\beta}$ will not be consistently estimated when there is serial correlation in the error term η_t .

The choice of the variables in X_t , used in the *ex post* real rate regressions (with Canadian data) were identical to CM's choice. These included a constant term, a time trend, the nominal interest rate, and lagged inflation rates. We used (three month) interest rates on treasury bills. Various rates of inflation were calculated using the CPI (all items) and the Industrial Product Price Index (all commodities), expressed as annual rates.

II.3 Estimates of Short-Term Real Interest Rates

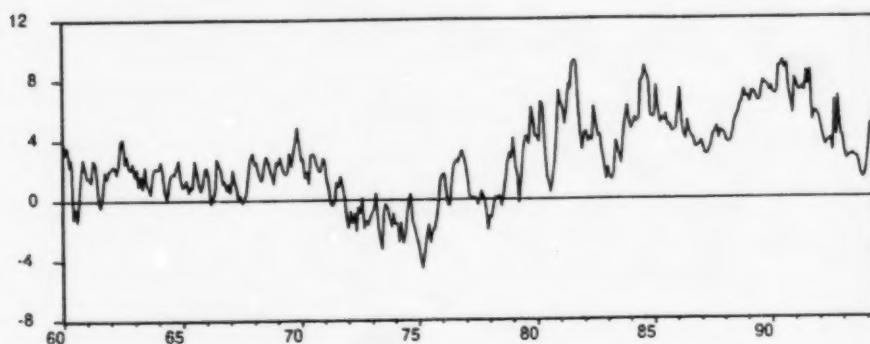
Decade averages of *ex ante* real rates are presented below, obtained from the *ex post* real rate regressions using the three month rate on Canadian treasury bills. The CPI was used in calculating the rate of inflation. Results were similar when the IPPI was used in the inflation rate calculations.

Table 1. Short-Term Real Rate Averages

Time Period	1960-1969	1970-1979	1980-1989	1990-1994
XATB Real Interest Rate	1.7311	0.2613	4.9914	5.3407

The series (which we refer to as XATB) accords with the standard historical characterisation of real interest rates (see Blanchard and Summers (1984)). Real interest rates were low (and at some times even negative) throughout the 1960s and 1970s, however in the 1980s, real rates increased dramatically. The above results suggest that the average level of *ex ante* real interest rates in Canada in the 1990s is higher than its 1980 average. If one were to look carefully at the behaviour of *ex ante* real rates in the 1990s (see Figure 1), one would notice that real rates were significantly high at the beginning of the decade and then fell sharply. Presently, *ex ante* real rates appear to be on the rise although their current level is below historical highs.

Figure 1. Short-Term Real Rate (XATB) Estimates, 1960-1994



The above results were obtained by using OLS without correcting for serial correlation in the error term. OLS yields consistent estimates of the coefficients in the *ex post* regressions, however, statistical inferences are not valid if test statistics are based on the variance-covariance matrix of $\hat{\beta}$. CM suggest an alternative diagnostic check of the adequacy of the X_t specification. Since the data are overlapping, and given the assumption of rational expectations, the forecast error $\varepsilon_{t,j}$ will follow a second order moving average process. If the relevant information for predicting the real rate is included in X_t , $u_{t,j}$ will be small and $\eta_{t,j}$ will have the same time series properties as $\varepsilon_{t,j}$. Ljung-Box and Breusch-Godfrey tests for serial correlation suggest that $\eta_{t,j}$ follows an MA(2) process (see appendix 1). Thus, the specification of X_t appears to be adequate.

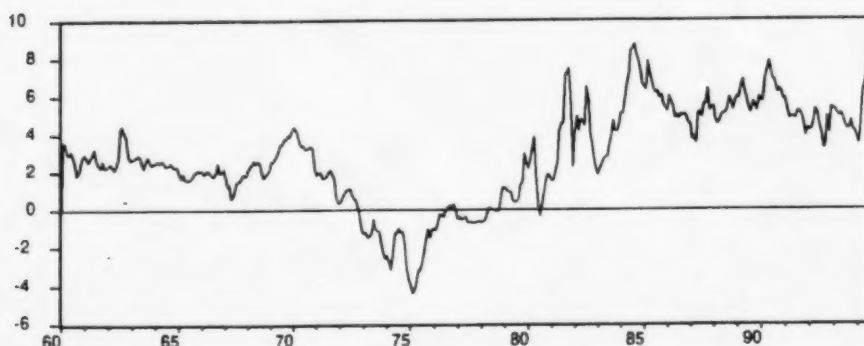
II.4 Real Rate Estimates using the Hodrick-Prescott Filtered Inflation Rate

One of the most widely used proxies of expected inflation is trend inflation as estimated by a Hodrick-Prescott (HP) filter. The HP filter identifies the permanent and transitory components of a series. In essence, the HP filter trades off goodness of fit between the trend and the actual series, against a degree of smoothness chosen for the trend component.² In constructing estimates of medium term real rates, we subtracted the HP filtered inflation rate (with a smoothness parameter of 1600) from the one-year yield on (benchmark) Government of Canada bonds. Our estimates are presented below. Throughout this paper we refer to this real rate series as XAHP. This series exhibits, to a large extent, the same pattern as the XATB series. Different price indices were considered though there was no significant difference in the final estimates.

Table 2. Medium-Term Real Rate Averages

Time Period	1960-1969	1970-1979	1980-1989	1990-1994
XAHP Real Interest Rate	2.4076	-0.0257	4.8023	5.2917

Figure 2. Medium-Term Real Rate (XAHP) Estimates, 1960-1994



II.5 The Barro - Sala-i-Martin Approach

One of the limitations of using the Cumby-Mishkin approach is that it is difficult to incorporate long-term interest and inflation rates. To some, the three month horizon adopted in the above approach may seem inane. The Barro - Sala-i-Martin (1990) (BM) approach allows us to circumvent the unnecessary restrictions on the length of the time horizon. Inflation in this case follows an ARMA process and thus the (log of the) price index follows an ARIMA process (when inflation is regarded as the first-difference of the

²Mang, 1994, p. 17.

log of the price level). Expected rates of inflation are based on forecasts of future levels of the consumer price index from the ARIMA model. The rate of expected inflation reflects the expected change in the log of the price index over a period of 5 years. Real rates are calculated simply by subtracting the forecasted value of future inflation from the nominal interest rate. The appropriate nominal interest rate in this case should be the yield on 5 year Government of Canada bonds, to maintain the correct maturity horizon however, this series does not extend past the early 1980s. Thus, we use the average yield on Government of Canada (5 to 10 year bonds) as a proxy.

As above, the period that we are interested in is 1960-1994. Expectations of future inflation are based on forecasts of rolling ARMA regressions. An ARIMA process is estimated from 1946:01 to 1960:01, using monthly data, for the (log of the) consumer price index. The (log) level of the CPI in 1965:01 is forecasted from this equation. Expected inflation over the period 1960:01-1965:01 is calculated by subtracting the log of the CPI in 1960:01 from the log of the (forecasted) CPI in 1965:01, expressing this rate of inflation as an annual rate requires us to divide by a factor of five. Calculating the *ex ante* real rate involves subtracting the forecast (made at 1960:01) from the nominal yield on a five year bond issued at 1960:01. The ARMA process is then estimated from 1946:01 to 1960:02, and a forecast is computed for the inflation rate at 1965:02. BM focus on short term real rates. Their forecasts are quarterly and they use the corresponding three month treasury bill rate to construct *ex ante* series. We have decided to extend the forecast horizon to five years and use long-term bond yields.

Over the period 1946:01 to 1994:09, inflation (calculated as month over month) can be characterized as an ARMA(1,1) process with a seasonal (monthly) moving average component, similar to BM although they impose deterministic seasonals in their representation. As well, Perron (1993) also finds this ARMA representation of the inflation process for Canada appropriate. Thus, the functional form for the inflation regressions is an ARMA(1,1,0,1),

$$(1 - \phi_1 L)\pi_t = \mu + (1 + \gamma_1 L)(1 + \gamma_2 L^{12})e_t. \quad (2.6)$$

Remarkably, parameter estimates remained stable across the series of rolling regressions (see appendix 2).³ Decade averages and estimates of *ex ante* real rates for 1960-1994 are presented below.

II.6 Estimates of Long-Term Real Rates

In contrast to the estimates of the *ex ante* real rate obtained from the Cumby-Mishkin approach, average real rates were higher in the 1970-1979 period and lower in

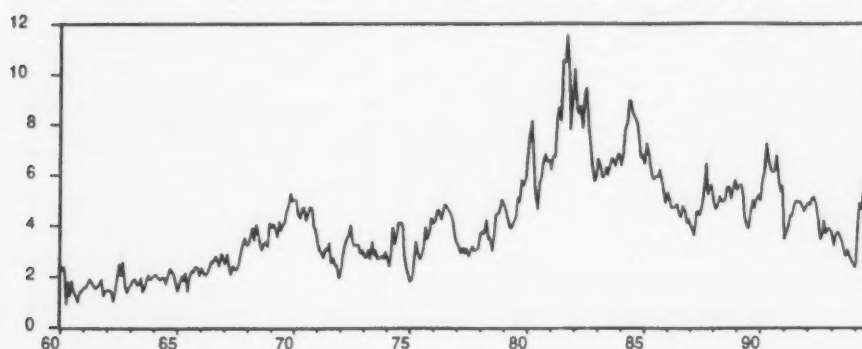
³Rolling ARMA regressions were also carried out where the estimation period was fixed at various lengths, however, parameter estimates over the sample period were extremely unstable.

the 1990-1994 period (see Table 3). However, real rates followed a similar pattern in the 1990s - high at the beginning of the decade and falling, rising in 1994. Current *ex ante* real rates are below their historical highs (see Figure 3). We refer to this series as XARM.

Table 3. Long-Term Real Rate Averages

Time Period	1960-1969	1970-1979	1980-1989	1990-1994
XARM Real Interest Rate	2.3710	3.5900	6.2928	4.5510

Figure 3. Long-Term Real Rate (XARM) Estimates, 1960-1994



Again, these estimates support the standard historical characterisation. Over the 1960-1994 period, the long-term *ex ante* real rate estimates were subject to greater swings in comparison to the short-term rate estimates. Real rate volatility increased after the 1960s - a characteristic shared by all the various estimates. Long-term real rates over the sample period did not dip below zero as was the case for the short-term XATB series. Estimates (based on all approaches) for 1994 range from 4 to 6 per cent. Real rates do appear to be on the rise, however, they are well below historical levels.

Section III

III.1 The Fisher Effect

A high correlation between interest rates and inflation is commonly referred to as the Fisher effect. It suggests that movements in *short-term* interest rates largely reflect changes in expected inflation and in turn, these rates have some predictive ability for

future inflation.⁴ Mishkin (1992) distinguishes between short and long-run Fisher effects. The short-run effect exists if a change in expected inflation is associated with a change in interest rates. If inflation and interest rates share a common trend when they exhibit trends, then a long-run Fisher effect is said to exist. The following section will examine the existence of the long-run effect.

Visual inspection of the *ex ante* real interest rate plots in the previous section suggest that the real rate might not be mean-reverting. There is a direct relationship between the existence of a long-run Fisher effect and the stationarity of the real interest, under the assumption of rational expectations. A long-run Fisher effect implies that inflation and nominal interest rates have a common stochastic trend when they exhibit trends, (*i.e.* they are cointegrated) from which it follows that the real rate is stationary. If inflation and interest rates are non-stationary over this period, we would expect to observe a strong Fisher effect if these series share a common trend. Failing to observe a strong Fisher effect would provide additional support for the hypothesis that the *ex ante* real rate is non-stationary.

Previously, evaluating the Fisher effect has involved testing the strength of the correlation between the level of current nominal interest rates and future inflation. A simple forecasting equation, where interest rates were regressed on future inflation (with the same horizon length) was the main approach used in examining the Fisher effect. The forecasting equation can be written as,

$$\pi_t^m = \alpha + \beta i_t^m + \eta_t \quad (3.1)$$

where

π_t^m = m -period future inflation rate from time t to $t+m$,

i_t^m = m -period interest rate known at time t .

Mishkin (1992) evaluates this methodology and argues that it does not provide reliable evidence--it is subject to the spurious regression phenomenon. Using U.S. data on one and three-month inflation and interest rates, Mishkin finds unit roots in both the right and left-hand-side variables in the above equation.

III.2 Testing for Unit Roots and Monte Carlo Simulations

Mishkin calculates various test-statistics such as the Dickey-Fuller and the Phillips modified t -statistic for the various inflation and interest rates. The critical values for these test-statistics, calculated by Dickey and Fuller, are not appropriate if the underlying time

⁴Mishkin, 1992, p. 195.

series models for inflation and interest rates are mixed processes, even for large sample sizes.⁵ In order to obtain the correct small sample distributions for these test-statistics, Mishkin performs Monte Carlo simulations. Following Mishkin's approach, tests for unit roots were conducted on Canadian three and six month inflation and interest rates over the 1960-1994 period, with the appropriate distributions for the Dickey-Fuller and the Augmented Dickey-Fuller test-statistics calculated. Calculating these test-statistics (for the series y , as an example) involves estimating the model

$$y_t = \alpha + \rho y_{t-1} + \sum_{i=1}^{p-1} \kappa_i \Delta y_{t-i} + u_t, \quad (3.2)$$

where the coefficient ρ should equal one if there is a unit root. The Dickey-Fuller t -statistic is $\tau = (\hat{\rho} - 1)/se(\hat{\rho})$.

The data generating processes for inflation⁶ and interest rates used in the Monte Carlo simulations were obtained from (mixed) ARIMA models. These models were estimated (with monthly observations) in first-differenced form, which assumes the existence of unit roots. The roots of the autoregressive and moving average polynomials fell outside the unit circle. Thus, the models will produce series that are stationary in first differences. The sample size used in the simulations corresponds to the time horizon mentioned above. Having generated the appropriate distributions for the test-statistics, we are able to calculate the marginal significance levels. The marginal significance level is the probability of getting that high a test-statistic or higher (in our case, a larger negative number) under the null hypothesis that the variable contains a unit root. A marginal significance level less than 0.05 indicates a rejection of the null hypothesis of a unit root at the 5% level.

Distributions for the Dickey-Fuller and Augmented Dickey-Fuller test-statistics were calculated. In calculating the ADF test-statistics, the lag lengths were chosen by two procedures. The first procedure is similar to Perron's (1990) method, where the lag length that produced a significant t -statistic (at the 5% level) on the last lagged differenced value was selected. In the second procedure, the ADF test-statistic distribution was computed with a fixed lag length of 12. Table 4 presents the DF and ADF test-statistics along with the marginal significance levels in parentheses.

⁵Schwert, 1987, p.74.

⁶The three and six month inflation rate series were calculated from the one month series.

Table 4. ADF Test-Statistics

m (months)	Test Statistics for π_t^m			Test Statistics for i_t^m		
	DF	ADF(P)	ADF(F)	DF	ADF(P)	ADF(F)
3	-5.2903 (0.061)	-3.6082 (0.055)	-1.6768 (0.400)	-1.6937 (0.314)	-2.1389 (0.219)	-1.8911 (0.339)
6	-2.9885 (0.116)	-2.7628 (0.107)	-1.4184 (0.228)	-1.8190 (0.221)	-2.3400 (0.146)	-2.0989 (0.535)

In the above table, DF is the Dickey-Fuller t-statistic, ADF(P) is the Augmented Dickey-Fuller t-statistic calculated by a procedure similar to Perron (1990), and ADF(F) is the Augmented Dickey-Fuller t-statistic calculated with a fixed lag length of 12.

Using the test-statistics for the six month inflation rate as well as those for the three and six month interest rates, we would not be able to reject the null hypothesis of a unit root using the MacKinnon critical values. The marginal significance levels suggest that the test-statistics (near the MacKinnon critical values) are further away from being significant than otherwise believed. In contrast, using the test statistics for the three-month inflation rate, we could easily reject the null hypothesis of a unit root (in some cases at the 1% level of significance) using the MacKinnon critical values, which would support stationarity. However, in this case, the Monte Carlo procedure prevents us from rejecting a true null hypothesis.

Using the appropriate small sample distributions generated by the Monte Carlo simulations, we find that the null hypothesis of a unit root *cannot* be rejected at a 5% level of significance for inflation and interest rates. Overall the results clearly suggest that short-term inflation and interest rate series are indeed non-stationary over the 1960-1994 period. Mishkin (1992) reaches the same conclusions using U.S. data on one and three month inflation and interest rates.

III.3 The Long-Run Fisher Effect and Real Interest Rates

There is a direct relationship between the (full) long-run Fisher effect and the stationarity of the *ex ante* real interest rate under the hypothesis of rational expectations. Recall that the existence of a long-run Fisher effect requires that inflation and interest rates have a common stochastic trend when they exhibit trends. If inflation and interest rates are $I(1)$, then they are said to be cointegrated (of order 1,1) if a linear combination of them is $I(0)$. In terms of the forecasting equation, inflation and interest rates (given that they are both $I(1)$) would be cointegrated if the error term (η_t) were $I(0)$. Cointegration of

π_t^m and i_t^m , which is what we mean by a long-run Fisher effect, implies that a linear combination of these variables is stationary.⁷

Two sets of cointegration tests (*i.e.* unit root tests on the error terms in the forecasting equation) were performed. The first cointegration test was a test of the *full* long-run Fisher effect. We tested for a unit root in $\pi_t^m - i_t^m$, under the assumption that the beta coefficient was equal to one. In this case according to Mishkin, inflation and interest rates move one-for-one in the long run. The second cointegration test was a test of the *partial* long-run Fisher effect, where we tested for a unit root in $\pi_t^m - \hat{\beta}i_t^m$.

The cointegration test mentioned above for the full long-run Fisher effect is related directly to a unit root test on the *ex ante* real interest rate. Defining the real rate as,

$$r_t^m = i_t^m - E_t \pi_t^m, \quad (3.3)$$

testing for a *full* long-run Fisher effect is equivalent to testing for a unit root in the *ex ante* real rate. Under rational expectations, actual and expected inflation differ by an unforecastable error term which is $I(0)$ or stationary in levels. Subtracting the forecast error from both sides of the equation above and rearranging yields,

$$\pi_t^m - i_t^m = \varepsilon_t - r_t^m. \quad (3.4)$$

From this equation and under the assumption of rational expectations, it is clear that the left-hand side of equation (3.4) can only be $I(1)$ if the real interest rate is also $I(1)$. Thus, the existence of a *full* long-run Fisher effect implies that the real rate is stationary.

Again the Monte Carlo procedure is used as an alternative to the Dickey-Fuller critical values, producing small sample distributions of the test statistics. The data generating processes for inflation and interest rates used in the above simulations were also used in the cointegration simulations. The data generating processes produced series that contained unit roots and were not (in terms of the full and partial Fisher effect equations) cointegrated. Distributions for the Dickey-Fuller and Augmented Dickey-Fuller test-statistics were computed, using the same procedures for choosing the lag length as mentioned previously. Table 5 presents the test-statistics along with the marginal significance level. Recall that a marginal significance level less than 0.05 indicates a rejection of the null hypothesis of no cointegration at the 5% level.

⁷ Mishkin, 1992, p.205.

Table 5. Cointegration Tests

	Test-Statistics for the Full Fisher Effect $(\pi_t^m - i_t^m)$			Test-Statistics for the Partial Fisher Effect $(\pi_t^m - \hat{\beta} i_t^m)^8$		
m (months)	DF	ADF(P)	ADF(F)	DF	ADF(P)	ADF(F)
3	-5.5184 (0.047)*	-4.1266 (0.027)*	-2.2333 (0.202)	-6.2053 (0.020)*	-4.4245 (0.017)*	-2.1141 (0.231)
6	-3.5820 (0.047)*	-3.7192 (0.027)*	-1.9332 (0.267)	-3.7754 (0.031)*	-3.7031 (0.028)*	-1.6607 (0.368)

In the above table, DF is the Dickey-Fuller t-statistic, ADF(P) is the Augmented Dickey-Fuller t-statistic calculated by a procedure similar to Perron (1990), and ADF(F) is the Augmented Dickey-Fuller t-statistic calculated with a fixed lag length of 12. An asterisk denotes a rejection of the null hypothesis at the 5% level.

From the results in Table 5, it is evident that rejection of the null hypothesis (of no cointegration) is sensitive to the choice of the test-statistic. The Dickey-Fuller and the Augmented Dickey-Fuller test-statistics (where the lag length is chosen endogenously, *i.e.* Perron's (1990) method) reject the null hypothesis at the 5% level of significance for the full and partial Fisher effects. When the lag length is fixed at 12, the null hypothesis cannot be rejected in either case.

Interestingly, Mishkin (1992) does not report (or mention) the Augmented Dickey-Fuller test statistics for the cointegration tests. He (as do we) finds that his results for the unit root tests on inflation and interest rates are not sensitive to (the procedure used to determine) the lag length, however, he does not state whether his results from the cointegration tests are sensitive to the lag length. The results in this section are similar to Mishkin's for the unit root tests on inflation and interest rates--these series are $I(1)$. As well, using the Dickey-Fuller test-statistics we find, as Mishkin does, that a full long-run Fisher effect does exist for the three month rates over a similar time horizon, although we are unable to make comparisons with regard to the Augmented Dickey-Fuller test-statistics.

Evidence in this section regarding stationarity of the short-term real interest rate is mixed. Up to this point we have only focused on short-term rates in order to illustrate the relationship between the Fisher effect and stationarity. In the next section we address the issue of stationarity from different perspectives, incorporating real rates of various horizons.

⁸The estimate of beta used in these calculations is based on the value from the Fama (1975) regression with Canadian data.

Section IV

IV.1 Stationarity Revisited

In Section II and III of our paper we examined various techniques used to calculate *ex ante* real interest rates and tested for the existence of the Fisher effect. The inconclusive nature of the test results for the Fisher effect raises questions about the time series properties of real interest rates. This section further explores the time series properties of real interest rates by examining the implications of non-stationarity and provides a brief discussion of the theoretical support for non-stationarity. Next, we attempt to quantify the relative contributions of different types of shocks to real rates.

Even a cursory review of the historical pattern of long-term *ex ante* real interest rates in Canada suggests that real rates may not be dynamically stable.⁹ In the early 1960s movements in real interest rates were sporadic, without any major swings in either direction, real rates fluctuated between one and two per cent. Toward the end of the decade and throughout the 1970s, real rates became increasingly volatile and averaged three percent. Real rates increased substantially in the 1980s, fluctuating wildly and reaching levels as high as 12 percent. This volatility has not diminished significantly through the 1990s, although real rates on average have declined in comparison to levels of the previous decade. Tests in the previous section of our paper provided mixed support for the hypothesis of stationarity with regard to short-term real rates (as calculated under the hypothesis of rational expectations). Unit root tests were performed on all our real rate series.¹⁰ Based on the results in appendix 3 we cannot accept the hypothesis of stationarity. We now shift our focus to analyse the nature of fluctuations in the real interest rate.

Unit root tests performed above and in appendix 3 have difficulty differentiating between a stationary series and a stationary series plus a small random walk.¹¹ To circumvent the low power of unit root tests (with regard to inferences about stationarity) Campbell and Mankiw (1987), in addition to Cochrane (1988), attempt to determine the importance of the permanent component of a time series. It has been argued that their measures of the permanent component are not adequate in an environment where the variable in question is subject to multiple disturbances.¹² Blanchard and Quah (1989) first identify (multiple) underlying disturbances and then proceed to quantify the importance of each disturbance to the behaviour of the series.

⁹See Figure 3 in Section II for example.

¹⁰Various Augmented Dickey-Fuller tests are presented in appendix 3.

¹¹Cochrane, 1988, p. 895.

¹²Quah, 1992.

Attempts at determining the importance of the permanent component to the behaviour of a variable, including both the univariate (Campbell-Mankiw and Cochrane) approach and the multivariate (Blanchard-Quah) approach, have largely focused on aggregates such as GNP. In this section we extend these approaches to include the real interest rate. Application of the univariate approach provides us with an estimate of the permanent component in the real rate series. Within this framework, however, we are constrained to a characterisation of the permanent component as a random walk in addition to, as mentioned above, a single disturbance affecting the real rate. The Blanchard-Quah approach permits us to address essentially the same issues in a richer framework.

IV.2 The Univariate Time Series Framework: Measuring the Importance of the Random Walk Component

If fluctuations in r_t are permanent, the underlying non-stationary time series process can be characterized as a *random walk*.

$$r_t = r_{t-1} + \varepsilon_t \quad (4.1)$$

Fluctuations are permanent in the sense that long-term forecasts of r will be affected by the same amount as the shock, at each date. Further, if we assume that r is first difference-stationary, we can express r_t as a combination of a random walk (permanent) component and a stationary series. This is commonly referred to as the Beveridge-Nelson (1981) decomposition. Consider the following first difference-stationary process with the moving average representation,

$$\Delta r_t = \sum_{j=0}^{\infty} \psi_j \varepsilon_{t-j} \quad (4.2)$$

We can decompose r into the permanent component z and the temporary component c ,

$$r_t = z_t + c_t, \quad (4.3)$$

where,

$$z_t = z_{t-1} + \left(\sum_{j=0}^{\infty} \psi_j \right) \varepsilon_t, \quad (4.4)$$

$$-c_t = \left(\sum_{j=1}^{\infty} \psi_j \right) \varepsilon_t + \left(\sum_{j=2}^{\infty} \psi_j \right) \varepsilon_{t-1} + \dots \quad (4.5)$$

There are two related ways to think about the relative importance of these components and their implications for long-term forecasts. Campbell and Mankiw evaluate the effect of a shock on long-term forecasts by estimating impulse response functions, while Cochrane measures the "size" of the random walk component.

IV.3 Campbell and Mankiw's Approach

When a time series is expressed in its infinite moving average form, the coefficients on the current and lagged disturbances represent the impulse response function. The relationship between long-term forecasts and impulse responses is direct. The magnitude of the impulse response at a sufficiently long lag length can be interpreted as the effect of a one per cent shock (to the variable described by the univariate time series model) on the forecast of that variable over the long term.

Estimating the magnitude of the impulses allows us to make inferences about the effects of shocks on long-term forecasts and consequently on the appropriateness of the process generating the variable in question. As an example, consider an unexpected change (*i.e.* a shock) in GDP of 1%. If long-term forecasts are changed by 1% after the shock is realized, it is highly likely that GDP is generated by a random walk process. In this case the value of the impulse response taken at a long lag length would be close to one. In contrast, if long-term forecasts are not affected by the shock at all, a (trend) stationary process is most likely the appropriate characterisation and the impulse response would be close to zero.

Campbell and Mankiw calculate impulse responses for GNP based on various low order ARIMA models of GNP. It is apparent (see appendix 4) that the estimates of the impulse responses will be dependent on the autoregressive and moving average parameter estimates from the ARIMA model. It has been argued¹³ that this method yields inaccurate results. The low order models used to determine the impulse responses capture only the short-run dynamics, exaggerating the size of the random walk component. We therefore apply an approach proposed by Cochrane which yields non-parametric estimates of the random walk component and impulse responses.

IV.4 Cochrane's Approach

Cochrane approaches the issue of whether fluctuations (in GNP) are permanent or transitory from a different perspective. The approach evaluates the importance of the random walk component to the behaviour of the series. Recall from the Beveridge-Nelson decomposition that a first difference-stationary time series can be expressed as a

¹³Cochrane, 1988, p.915.

combination of a random walk plus a stationary series. If the variance of the shocks to the random walk component is zero, long-term forecasts are not responsive to shocks. However, if this variance is equal to the variance of the first difference of the series, long-term forecasts are affected by the amount of the shock at each date. Consider the random walk model for the real interest rate in equation (4.1). Solving (4.1) backwards yields,

$$r_t = \sum_{i=1}^k \varepsilon_{t+i-1} + r_{t-k}. \quad (4.6)$$

Now, take the variance of the '*k*-difference',

$$\text{var}(r_t - r_{t-k}) = \sum_{i=1}^k \text{var}(\varepsilon_{t+i-1}) = k \cdot \sigma_\varepsilon^2. \quad (4.7)$$

As *k* approaches infinity, the *k*-difference variance also approaches infinity. In contrast, if the transitory component of *r* is responsible for the fluctuations in the real rate, the variance of its *k*-difference approaches a constant.

$$\text{var}(r_t - r_{t-k}) = 2 \cdot \sigma_r^2 \quad (4.8)$$

Next, Cochrane derives the relationship between the autocorrelation coefficients of Δr_t (ρ_j), and $(1/k)\text{var}(r_t - r_{t-k})$ denoted σ_k^2 .

$$\sigma_k^2 = \left(1 + 2 \sum_{j=1}^{k-1} (1 - j/k) \rho_j \right) \sigma_{\Delta r}^2 \quad (4.9)$$

Recall the (permanent) random walk component from equation (4.4). The variance of the shocks to the random walk component can be expressed as,

$$\sigma_{\Delta r}^2 = \left(\sum_{j=0}^{\infty} \psi_j \right)^2 \sigma_\varepsilon^2. \quad (4.10)$$

As *k* approaches infinity, the limit of σ_k^2 approaches (4.10).

$$\lim_{k \rightarrow \infty} \sigma_k^2 = \left(1 + 2 \sum_{j=1}^{\infty} \rho_j \right) \sigma_{\Delta r}^2 = \sigma_{\Delta r}^2 \quad (4.11)$$

Cochrane's variance ratio is $\sigma_k^2 / \sigma_{\Delta r}^2$. The greater is the importance of the random walk component to the behaviour of the *r*, the closer this ratio will be toward one, as *k* increases. In this case, fluctuations are permanent. If this ratio is zero, the random walk component is not responsible for the fluctuations in *r*.

Rose (1988) applies Cochrane's approach to measuring the importance of the unit root component in various U.S. *nominal* interest rates. In his model, Rose assumes that the real interest rate is non-stationary and forms the random walk component, while the stationary component consists of inflationary expectations. By construction, the variance ratio tests which Rose performs, provide us with a measure of the importance of the random walk component in terms of the nominal interest rate. The results suggest that a high proportion of the innovation variance is due to the random walk component. Rose simply attributes this variation to the real interest rate.¹⁴

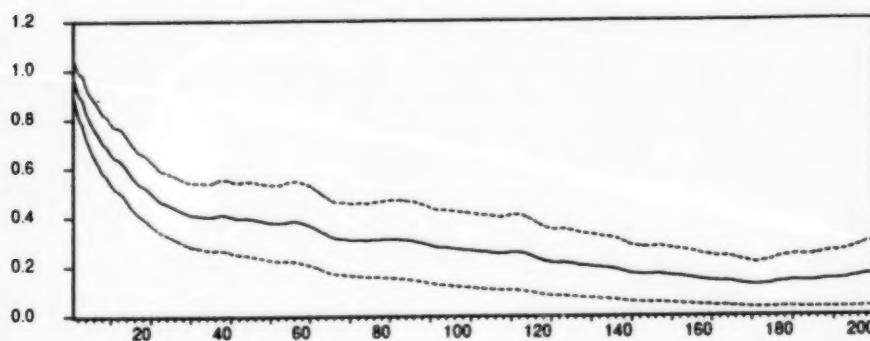
Applying Cochrane's approach directly to our real interest rate series permits us to evaluate the "size" of the random walk component in the real rate without simply defining this rate as a random walk process. The formula used in computing estimates of the variance ratio is given in Rose (1988) and Campbell-Mankiw (1987) as,

$$\hat{V}^k \equiv \frac{\hat{\sigma}_{\Delta r}^2}{\hat{\sigma}_{\Delta r}^2} = 1 + 2 \sum_{j=0}^k \left(1 - \frac{j}{(k+1)} \right) \hat{\rho}_j \quad (4.12)$$

where $\sigma_{\Delta r}^2$ is the variance of the first-differenced series of the real interest rate. Paquet (1994) suggests that increasing k up to half the sample size is appropriate.¹⁵

Figure 4 below shows a plot of the variance ratio¹⁶ for values of k up to half of the sample size (appendix 5 contains other plots based on alternative real interest rates). The solid line represents $\hat{V}^k$ and the broken lines are confidence intervals (*i.e.* $\hat{V}^k \pm \sigma_{\hat{V}^k}$) constructed from the asymptotic standard errors.

Figure 4. Variance Ratio Estimates for XARM



¹⁴Gokey (1990) also argues that Rose (1988) used incorrect inferential procedures. See Garcia and Perron, 1994, footnote 2.

¹⁵Paquet, 1994, p.3-35.

¹⁶The calculations are based on monthly data over the period 1960-1994 using the long-term *ex ante* real rate series XARM.

It is apparent from the graph that the plot settles down at approximately 0.2. This suggests that the permanent component of the real interest rate is responsible for a relatively small proportion of the variance in Δr_t . These findings are robust with regard to the alternative real interest rate series and data frequency. In terms of long-term forecasts of the real interest rate, forecasts do not move one-for-one with shocks at each date. Since the variance ratio falls between zero and one, an appropriate characterisation of the behaviour of the real interest rate might be a difference-stationary ARMA process with a small random walk.¹⁷

The program that calculated the variance ratio estimates also computed the impulse response at each k . Values of the impulse response are presented in appendix 6. The results range from 0.40 to 0.46 with monthly data. This suggests that, following a shock to the real interest rate, long-term forecasts of the real interest rate move about two-fifths of the amount of the shock.

IV.5 Theoretical Support for Unit Root Behaviour in the Real Interest Rate

The random walk literature is rich with various tests for non-stationarity and excessive volatility, however, theoretical support and insight into the existence of unit roots is usually absent. In this section we briefly review a variant of Taylor's (1980) model of overlapping wage contracts which produce unit root or *near* unit root behaviour in the real interest rate.

In West's (1988) model, the source of unit root behaviour in the real interest rate as well as real output is the monetary authority's policy of smoothing movements of (nominal) interest rates; the nominal interest rate follows a near random walk. Smoothing interest rates requires the monetary authority to follow a money supply rule¹⁸, with $0 \leq \lambda < 1$,

$$i_t = \lambda i_{t-1} + u_t \quad (4.13)$$

where, if the current nominal interest rate i_t is above (below) its target level, the money supply is increased (decreased). West abstracts from supply and demand shocks, and emphasises the role of monetary policy in output fluctuations. Serially uncorrelated shocks u_t enter the model through the money supply rule.

The basic idea underlying West's model is that the behaviour of the real interest rate follows from the serial correlation in the money supply rule. The root of $(1 - \lambda L)$ in

¹⁷Cochrane, 1988, p.916.

¹⁸This rule described in West (see appendix), 1988.

the money supply rule above carries over into the wage and price processes. Serial correlation in wages and prices results not only from the root in the money supply rule, but also from the existence of overlapping contracts.¹⁹ The nominal interest rate does not move instantaneously or symmetrically with expected inflation.

As a result of the serial correlation in the money supply (λ close to 1) and the additional persistence resulting from the overlapping contracts, movements in the real interest rate resemble *near* unit root behaviour. Near unit root behaviour (in terms of the real rate) is also observed in DeLong and Summers' (1986) variation of Taylor's (1979) model. In this model the source of the unit root behaviour is non-stationary shocks to aggregate demand.²⁰

IV.6 Criticisms and Remedial Measures

The approaches above attempt to estimate the importance of the permanent component to the behaviour of the real interest rate. They enable us to draw conclusions about the nature of fluctuations in the real rate. These conclusions, however, are based on the assumptions that either there is only one type of disturbance affecting the real interest rate (as in Campbell and Mankiw's approach) or that the permanent component takes on the structure of a random walk process (see Cochrane's approach). If we believe that there are multiple disturbances to the real interest rate or that the permanent component does not necessarily follow a random walk, the above approaches cannot identify the magnitude of the permanent component.²¹

Quah (1992) demonstrates that the fluctuations of an integrated series are not necessarily permanent, they can be dominated by transitory disturbances. The weakness, as Quah points out in the univariate approach, is that the sources of a variable's fluctuations are not identified. Without some restrictions, univariate time series models cannot meaningfully contribute in the determination of the relative importance of the permanent and transitory components. Blanchard and Quah (1989) adapt a multivariate approach which identifies the source of disturbances. Furthermore, by imposing some long-run restrictions they are able to quantify the relative contribution of each shock at various horizons. This approach enables us to ask more interesting questions in terms of the nature of fluctuations in the real interest rate.

¹⁹Cochrane, 1988, reiterates Taylor's assertion that overlapping contracts can be an endogenous source of persistence.

²⁰Rose, 1988, p.1107.

²¹Quah, 1992, p.107.

IV.7 Blanchard and Quah's Approach

First we assume that there are two kinds of disturbances uncorrelated with each other and that neither disturbance has a long-run effect on the unemployment gap ($U_t - U_t^*$), where U_t^* is the natural rate of unemployment.²² The first disturbance has only temporary effects on the real interest rate, while the second disturbance has permanent (long-run) effects.²³ These assumptions are sufficient to just identify the two disturbances.²⁴ We label disturbances having temporary effects on the real interest rate as nominal disturbances and those having permanent effects as real disturbances. The motivation for this interpretation is provided following the outline of Blanchard and Quah's approach.

We can represent this framework in the following form,

$$X_t = \sum_{i=0}^{\infty} A_i \varepsilon_{t-i}, \quad (4.14)$$

$$X_t = A(L)\varepsilon_t, \quad (4.15)$$

where $X_t = (\Delta r_t, (U_t - U_t^*))'$, and $\varepsilon_t = (\varepsilon_t^n, \varepsilon_t^r)'$. The elements in the X vector represent the change in the real interest rate and the unemployment gap respectively. The ε vector contains the nominal and real disturbances. We impose restrictions on $[A]$ in accordance with our characterisation of nominal and real disturbances. This means that the sum of the upper left-hand elements of $[A]$, denoted a_{11i} is zero.

$$\sum_{i=0}^{\infty} a_{11i} = 0 \quad (4.16)$$

As well, the variance-covariance matrix is equal to the identity matrix. This normalises the variance of the disturbances to be one and imposes orthogonality.

$$\text{Var}(\varepsilon) = I. \quad (4.17)$$

These assumptions will allow us to identify the disturbances, recovering them from a vector autoregression. The orthogonality condition permits us to examine the impact of each disturbance *ceteris paribus*.

²²The natural rate of unemployment time series was obtained from the CEFM series "LFNUR".

²³The results presented here obtained from the Blanchard and Quah exercise, are based on the real interest rate series XARM. Appendix 9 contains results based on the real rate series XATB and XAHP.

²⁴Blanchard and Quah, 1989, p.656.

Equations (4.14) to (4.17) define our model. We are able to recover this representation from the data by first estimating a VAR and then converting it into our model. Consider a VAR(k) representation of X_t ,

$$\begin{aligned} X_t &= B_1 X_{t-1} + \dots + B_k X_{t-k} + e_t, \\ \text{Var}(e) &= \Omega. \end{aligned} \quad (4.18)$$

We can invert (4.18) in the usual manner and obtain its infinite MA representation,

$$X_t = e_t + D_1 e_{t-1} + D_2 e_{t-2} + \dots \quad (4.19)$$

Recall our model expressed in a similar form,

$$X_t = A_0 \varepsilon_t + A_1 \varepsilon_{t-1} + A_2 \varepsilon_{t-2} + \dots \quad (4.20)$$

Using the impulse responses, we can recover the nominal and real disturbances from the residuals of the VAR. The residuals and the disturbances are related by,

$$\varepsilon_t = A_0^{-1} e_t \quad (4.21)$$

and the relationship between the impulse responses (with respect to the VAR) and the A coefficients in (4.20) is given by,

$$A_i = D_i A_0. \quad (4.22)$$

It is immediately apparent that the A_0 matrix is crucial in recovering the disturbances and impulse responses of our model. Given our assumptions, we are able to identify this key matrix. First we normalise the variance of the disturbances to be one, $A_0 A_0' = \Omega$, which identifies three elements of A_0 . This restriction also constrains the disturbances to be orthogonal. Next we impose the restriction that nominal disturbances have no long-run effects on the level of the real interest rate. These restrictions can be written as system of non-linear equations (see appendix 7).

Interpreting the disturbances in this type of model as nominal and real disturbances is, as Blanchard and Quah suggest, perilous. In contrast to many structural VAR models, we do not provide a single model which delivers the motivation for our framework. Thus, the inclusion of real interest rates and (actual and natural) unemployment rates in a bivariate VAR might seem somewhat arbitrary. However, by including these variables, we can exploit additional information about real interest rate fluctuations. There is, nevertheless, theoretical support for our framework.

Recent work²⁵ has extended the partial-equilibrium (Friedman) view of the labour market to include the effects of other prices, such as the real interest rate, on equilibrium in the labour market. Phelps (1994) finds support for this relationship, however, he concentrates on nonmonetary mechanisms and forces. Changes in the real interest rate affect the demand for labour, which influences the equilibrium unemployment rate. We believe that there are important linkages between real rates and unemployment rates, although not necessarily those outlined in Phelps' framework. As Gamber and Joutz (1993) along with Lippi and Reichlin (1993) observe, long-run characteristics can be obtained from a wide array of models.²⁶

A criticism of Phelps' model partially provides the motivation for our work. Many economists fear that Phelps' model is merely capturing the effect of tight money, which drives up real rates as it drives up unemployment.²⁷ By definition, Phelps excludes the influence of monetary forces. In our framework, monetary policy enters as a nominal disturbance, having only a temporary (though possibly a somewhat prolonged) effect on real rates. Tight money initially affects the nominal interest rate, however, in the longer-run, inflationary expectations move in such a way that there is no long-run impact on the level of the real interest rate. This characteristic can be attributed to a number of (neo-) classical models.²⁸ Tight money, by raising real rates, affects interest sensitive sectors of the economy. As aggregate demand falls off, firms reduce employment levels, though, it is also possible that these nominal disturbances could affect the natural rate of unemployment as well. In the long-run, however, the real rate falls back to its initial level and the unemployment gap disappears.

Additional motivation for our framework originates from structuralist models. Real disturbances such as technological improvements "pull" the employment rate up, thus lowering the unemployment rate.²⁹ These technological improvements also put upward pressure on the real rate by permanently increasing the (marginal) productivity of capital. It is possible that real disturbances could affect the natural rate of unemployment as well, however, these changes would likely lag behind those in the actual rate of unemployment, creating an unemployment gap. A variety of models of hysteresis in labour markets or models of the endogenous growth type would in principle not exclude an impact of these disturbances (both real and nominal) on the actual or natural unemployment rate even in the long run. The assumption that the unemployment gap is not affected in the long run does therefore not exclude a change in U^* . The unemployment gap itself, however, should be assumed to remain unchanged in the long run regardless of the type of

²⁵See Phelps' *Structural Slumps* (1994).

²⁶Gamber and Joutz, 1993, p.724. Lippi and Reichlin, 1993, p.646.

²⁷Phelps, 1995, p.229.

²⁸See Prachowny, 1991, p.133.

²⁹Phelps, 1994, p.60.

disturbance. Support for this assumption is based on the stationarity of the unemployment gap (see appendix 8).

The impulse responses of the model (from equation (4.20)) illustrate the dynamic effects of the real interest rate and the unemployment gap to each disturbance (of unit size). In the following figures, the vertical axes represent the level of the real interest rate and the unemployment gap (both expressed in percentage terms). The horizontal axis denotes time in quarters. Impulse responses are represented by the solid lines and the broken lines represent one standard deviation bands around the point estimate.³⁰

Figure 5. Real Rate Response to Nominal Disturbances

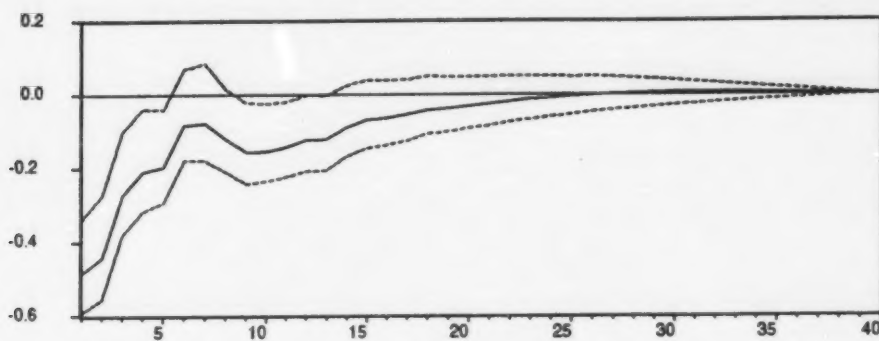
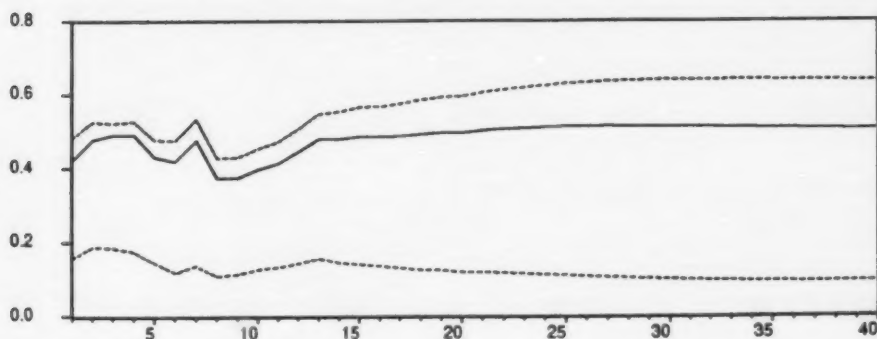


Figure 6. Real Rate Response to Real Disturbances



³⁰The bands around the point estimate were constructed by adding and subtracting the square root of the mean squared deviation (obtained from 1000 bootstrap replications) from the point estimate. As Blanchard and Quah (1989) observe, these bands need not be symmetric.

Figure 7. U-Gap Response to Nominal Disturbances

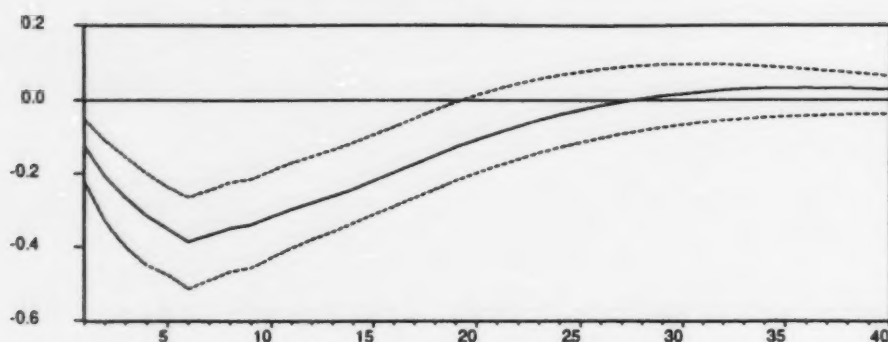
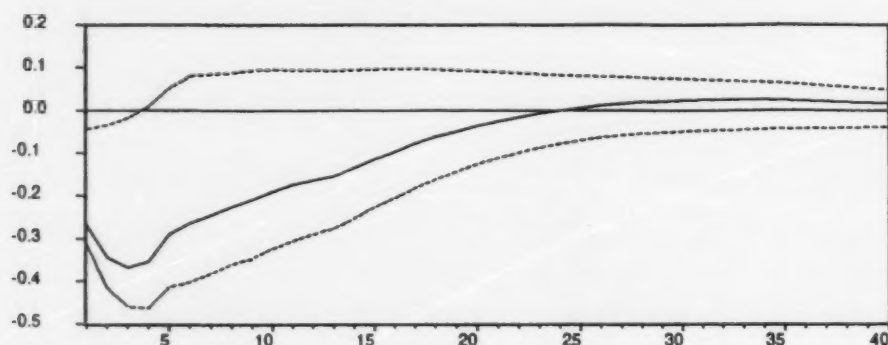


Figure 8. U-Gap Response to Real Disturbances



The impulse responses in Figures 5 and 6 accord with our expectations of the behaviour of the real interest rate in response to the two types of disturbances. In response to a positive nominal shock, such as an increase in the money supply, the real rate falls initially, however, in the long run (after 40 quarters) there is no permanent change. The effect on the real interest rate becomes very small after about 6 quarters, while the reduction in the unemployment gap lasts somewhat longer, up to six years. Responding to a favourable real shock, the real rate rises immediately following the shock, permanently raising the real rate. This effect cumulates over time, reaching a plateau after four years. The impulse response in Figures 7 and 8 accord with our theoretical motivation. In response to both shocks, the unemployment gap falls (*i.e.* the actual rate of unemployment is below the natural rate) in the short run, although returns to its initial level after approximately 40 quarters.

The impulse responses presented above provide a description of the dynamic effects of real and nominal disturbances. Next, we attempt to examine the relative contribution of each disturbance to fluctuations in the real interest rate as well as the unemployment gap. To quantify the relative contributions, we calculate variance decompositions for the real rate and the unemployment gap at various horizons. Calculating variance decompositions involves computing the variance of the forecast error

accounted for by a particular disturbance at various horizons.³¹ Blanchard and Quah define the k quarter-ahead forecast error as the difference between the actual value of the real rate (or unemployment gap) and its forecast from equation (4.19), k quarters earlier. The table below presents the point estimates of the variance decompositions along with their one-standard error bands.³² Impulse responses and variance decompositions for the short and medium term real rates are presented in appendix 9.

Table 6. Variance Decomposition - Percentage of the Expected Squared Forecast Error Produced by Nominal and Real Disturbances

Horizon	Real Interest Rate		Unemployment Gap	
	Nominal	Real	Nominal	Real
1	57.0 (37.5, 92.4)	43.0 (7.6, 62.5)	17.8 (6.5, 62.5)	82.2 (37.5, 93.5)
2	51.6 (32.9, 88.4)	48.4 (11.6, 67.1)	23.6 (11.3, 67.7)	76.4 (32.3, 88.7)
3	43.9 (27.2, 81.5)	56.1 (18.5, 72.8)	28.5 (15.4, 71.5)	71.5 (28.5, 84.6)
4	38.3 (23.2, 75.8)	61.7 (24.2, 76.8)	33.8 (19.8, 75.0)	66.2 (25.0, 80.2)
8	27.6 (16.6, 62.6)	72.4 (35.8, 82.5)	51.5 (34.2, 83.8)	48.5 (16.2, 65.8)
12	23.5 (13.3, 56.7)	76.5 (43.3, 86.7)	57.3 (38.1, 85.8)	42.7 (14.2, 61.9)
40	7.3 (4.4, 19.6)	92.7 (80.4, 95.6)	61.0 (40.8, 86.8)	39.0 (13.2, 59.2)

Note: The numbers in parentheses represent the one-standard deviation confidence bands obtained from the bootstrap simulation procedure.

Point estimates of the variance decomposition suggest that nominal disturbances over the (very) short-term horizon account for the majority of fluctuations in the real interest rate, while over the medium term, real disturbances are responsible for most of the fluctuations in the real rate. As the horizon increases, our identifying assumptions impose only one restriction, namely that the contribution of real disturbances tends to 100 per

³¹Gamber and Joutz, 1993, p.740.

³²These bands were calculated in the same manner as those for the impulse responses above.

cent. The large standard deviation bands seem to be symptomatic of the Blanchard and Quah (1989) approach. Estimates also suggest that fluctuations in the unemployment gap, over the short and medium term, are largely due to real disturbances while over the long term, their importance decreases. The estimates of the relative contributions of each disturbance, in terms of the unemployment gap, are not as striking as those for the real interest rate. There is a great deal of overlap between the confidence bands for both types of disturbances even at the 40 quarter horizon. Thus we cannot confidently attribute (the majority of) fluctuations in the unemployment gap to either disturbance, across short to long-term horizons.

IV.8 Real and Nominal Components of the Real Interest Rate

Using our structural VAR model we are able to decompose the movements in real rates and the unemployment gap into nominal and real components. We form the nominal component series of each variable by constructing the time path which would have been realised in the absence of real disturbances. Likewise, we form the real component series by constructing the time path which would have been realised in the absence of nominal disturbances. In this section we focus on long-term real rates³³ (the XARM series). Recall the underlying assumptions of our model; nominal disturbances do not have a long-run effect on the level of the real rate nor the unemployment gap while real disturbances can affect the real rate in the long-run, but not the unemployment gap. Thus, we would expect the real component series (of the real interest rate) to follow closely the (actual) real interest rate series. Figures 9 to 12 present the component series (in broken lines) and the actual series (in solid lines).³⁴ In all figures the actual series presented represent deviations from their respective means.³⁵

³³Component series for XATB and XAHP (not presented here) are similar to those for XARM. Graphs of these series are available upon request.

³⁴Nominal and real component series could not be constructed over the entire sample period. Constructing the series involved computing the nominal and real components from an impulse response function. This required a significant number of lagged disturbances.

³⁵The real rate and unemployment gap series were de-meanned prior to estimation following Blanchard and Quah (1989).

Figure 9. The Nominal Component of the Long-Term Real Rate

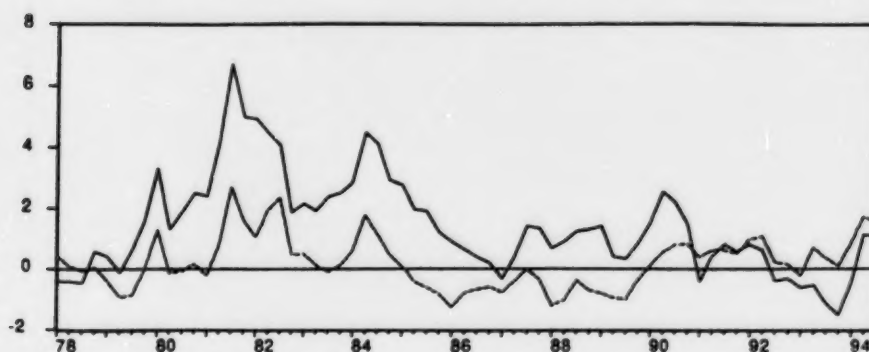
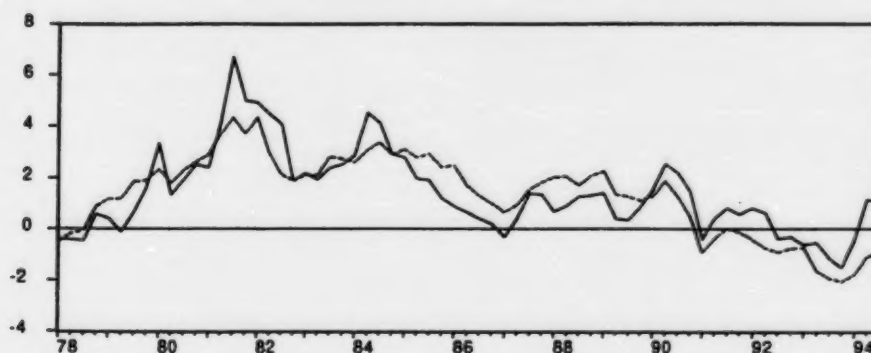


Figure 10. The Real Component of the Long-Term Real Rate



As expected, the real component series (Figure 10) follows (to a large extent) the actual series. This finding provides an insight into the puzzling economic phenomenon that over the second half of the 1980s real interest rates reached their highest levels, yet economic growth remained relatively strong for some time. The real component of the real interest rate over the 1985-1990 period accounts for almost all of the real interest rate fluctuations whereas the nominal component (in Figure 9) contributes modestly, if at all to movements in the real interest rate over this period. However, at the beginning of 1994 the nominal component series was closer to the actual series. The nominal and real component series move in a relatively synchronous fashion over the 1978-1994 period.

Figure 11. The Nominal Component of the Unemployment Gap

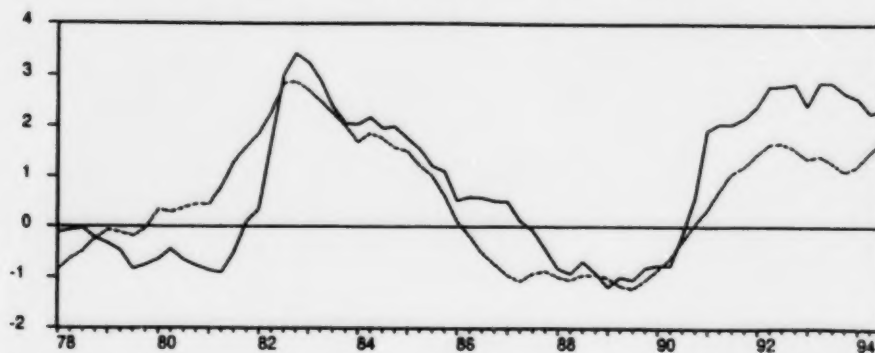
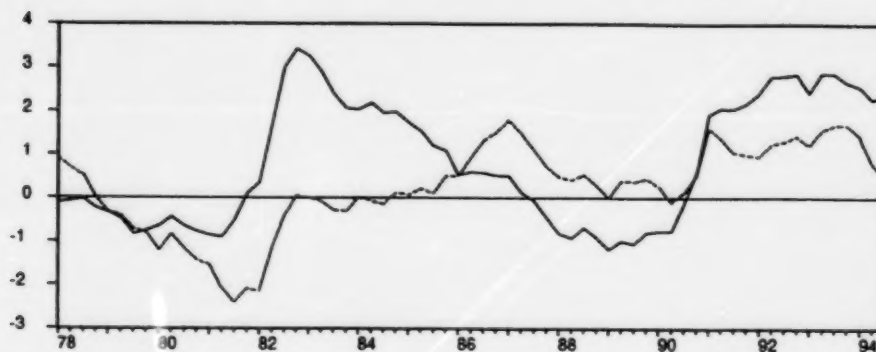


Figure 12. The Real Component of the Unemployment Gap



Figures 11 and 12 depict the component series for the unemployment gap ($U_t - U_t^*$). Interestingly, the nominal component series closely follows the actual series. Based on these figures, the recession in the early 1980s appears to have been caused by negative nominal disturbances while the cause of the 1990 recession could be attributed to adverse real *and* nominal disturbances. This provides a plausible explanation for the severity (in terms of prolonged labour market disequilibrium) of the recent recession. We also find an interesting relationship between the nominal components of the real rate and unemployment gap series. In Figure 13, the broken line represents the nominal component of the real interest rate and the solid line represents the nominal component of the unemployment gap series.

Figure 13. Nominal Components of the Real Rate and Unemployment Gap

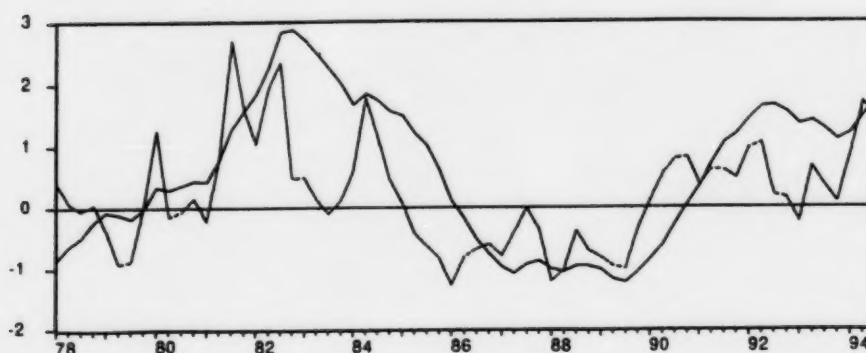


Figure 13 reveals a striking relationship between the movements in the nominal real rate component and the nominal unemployment gap component. Negative nominal disturbances resulted in increases in the nominal real rate component. These movements precede increases in the nominal unemployment gap component by an amount of time that seems consistent with the delayed effects of monetary policy. There also appears to be a strong correlation between the size of the spike in the nominal real rate component and the deviation of unemployment from its natural rate. However, one should exercise caution in interpreting the results in this manner since we are examining each component in isolation.

IV.9 Fluctuations in Real Interest Rates: Summary

Within a univariate framework we measured the importance of the random walk component and estimated the response of long-term forecasts to shocks in the real rate. Our results are mixed in the sense that fluctuations in the real rate were not wholly permanent nor wholly transitory. The random walk component is modest in size, accounting for approximately 20% of the variation in (the first-difference of) the real interest rate. And, long-term forecasts of the real rate move approximately 40% of the amount of the shock to the real rate.

The results obtained within the univariate framework are subject to a number of caveats. First, within the framework we were restricted to a single disturbance whose source was not identified. Secondly, the permanent component of the real rate was assumed to be a random walk. By adopting a multivariate framework, we were able to examine essentially the same issues although in a richer context. By distinguishing between disturbances that had temporary and permanent effects on the real interest rate, we measured the importance of each type of disturbance to the behaviour of the real rate at various time horizons. We found that disturbances having temporary effects (referred to as nominal disturbances) accounted for the majority of fluctuations in the short-run while those having permanent effects (referred to as real disturbances) accounted for almost all of the fluctuations in the real rate in the long run. After decomposing the

fluctuations of the real rate into nominal and real components, we constructed historical time paths for the various real rates that would have been realised in the absence of each type of disturbance.

Section V

V.1 Conclusions

Our estimates of real interest rates for Canada appear to accord with the general historical consensus. Real rates were low and relatively stable over the 1960s, while in the 1970s and 1980s, real rates bottomed out and then reached their historical highs. Across the short to long-term maturities, real rates in the 1990s were lower on average than those in the 1980s, however, in 1994 there was a significant upward trend.

Our results support Mishkin's (1992) finding that unit root tests on inflation and nominal interest rates cannot reject the hypothesis that these series are integrated to the order one. As well, using the Dickey-Fuller test-statistics we find, as Mishkin does, that a full long-run Fisher effect does exist for the three month rates over a similar time horizon, although we are unable to make comparisons with regard to the Augmented Dickey-Fuller test-statistics. Evidence regarding stationarity of the short-term real interest rate is mixed using Mishkin's methodology. We examined the Fisher effect as a topic to illustrate the connection between the Fisher effect and real interest rates and as an introduction to the issue of real rate non-stationarity.

Standard unit root tests, such as the DF and ADF test are known to have low power. Furthermore, the underlying data generating process - for the real interest rate as well as its determinants - might not be purely autoregressive. Monte Carlo experiments have been performed which allow for a richer ARIMA representation. Small sample distributions of conventional test-statistics as critical values have been calculated and the evidence for real interest rates of different horizons support the hypothesis of non-stationarity.

We employed alternative approaches to address the issue of stationarity. By examining the fluctuations of long-term forecasts and measuring the importance of the random walk component in the real interest rate, we were able to present a more accurate characterisation of the time series model generating the real rate within a univariate time series framework. Our results indicated that the random walk component was modest in size and that long-term real rate forecasts were sensitive to disturbances.

Recognising the limitations of this framework, we extended our analysis to a multivariate time series framework which allowed us to identify and quantify the effects of different types of disturbances to real interest rates. Disturbances which we characterised as nominal shocks were found to be important in explaining fluctuations in real rates in the immediate and short run while real shocks accounted for most of the fluctuations in the

medium and long run. Constructing time paths for real interest rates (paths which would have been realised in the absence of each disturbance) revealed a striking relationship between movements in nominal components which seem to be consistent with the delayed effects of monetary policy. Finally, our decomposition of the real interest rate and unemployment gap into real and nominal components offers plausible explanations for economic developments during the 1980s and 1990s.

V.2 Implications for Policy and Forecasting

This paper's assertion that the *ex ante* real interest rate is not stationary has implications for determining the effects of monetary and fiscal policies which may be significant.³⁶ Most of the literature dealing with real rate non-stationarity, to the best of our knowledge, fails to make explicit the implications for instability in real interest rates, although there is unanimous consent that these implications are indeed significant. If the real rate does play an important role in the transmission of macroeconomic policy then disturbances unrelated to policy could have long-lived effects on real interest rates in addition to affecting long-term real rate forecasts. This would likely obstruct the real interest rate channel through which monetary and fiscal policy influence the economy and in turn reduce the efficacy of policy actions.

Policy itself (if we think of it as a disturbance in the univariate time series model or a real shock in the multivariate model) could have adverse long-lived effects on the real rate and real rate forecasts. In order to avoid this scenario, policy actions and the commitment to them, should be clear and credible. This would help reduce the uncertainty faced by economic agents by improving their information set upon which they base their savings and investment decisions and keep the channel clear for future policy action.

Based on our results, it would be misleading to discuss the "appropriate" level of real interest rates in terms of a historical average or make guesses about their future levels based on whether current levels are above or below such an average. Our results indicate that an appropriate forecast of future real rates should be based on their most recent level, however, in some cases it might be more appropriate to ignore nominal fluctuations in current real rates and use the current "real" level. The current³⁷ real component value (abstracting from nominal disturbances) represents the level of the real component of the long-term (XARM) real interest rate in 1995:2 (1995:1 for the short-term (XATB) rate). Below we provide forecasts for short and long-term real rates based on their current values and current real component values.

³⁶Rose (1988) and Garcia and Perron (1994) also mention some implications for issues relating to finance theory.

³⁷These estimates are based on data from 1995:2.

Table 7. Forecasts of Short and Long-Term Real Interest Rates

Real Interest Rate	Forecast A (Current Value)	Forecast B (Current Real Component Value)
Short-term Real Rate	6.2035	4.2663
Long-term Real Rate	3.3716	4.1695

V.3 Further Work

This paper has constructed various *ex ante* real interest rate series and examined their time series properties. We have attempted to address the issue of non-stationarity from different perspectives through, what we believe, is a natural progression of econometric applications traditionally applied to other important macroeconomic variables. Many of the criticisms and weaknesses of our approaches are noted in the papers cited above and, rather than summarise these details, we refer readers to those articles. However, there are some additional issues which we would like to briefly comment on in this section.

Using the Fisher (1930) equation as a working definition of the real interest rate involves unobservable expectations of future inflation. Most of the approaches (such as the rolling ARIMA forecasting procedure) used to construct series of inflationary expectations are backward-looking. This makes it difficult for announcements regarding future policy changes to be incorporated into expectations.³⁸ Also, our sample period covers a significant number of different monetary and exchange rate regimes that have not been explicitly taken into account. Nevertheless, we believe that our XARM real rate estimates accurately reflect actual long-term *ex ante* real interest rates. In spite of our neglect of regime changes, our coefficient estimates remained relatively stable over the forecasting period for the long-term series.

Further work should examine the underlying determinants of the real and nominal real interest rate component series. For example, we might expect some instruments or indicators of monetary policy to be correlated with the nominal component series rather than the real component series. We could also extend this approach to examine issues related to the Ricardian equivalence hypothesis. Recent papers from the OECD³⁹ decompose long-term real interest rates into low and high frequency economic factors. We would also like to examine the relevance of their factors to our real and nominal component series.

³⁸Ragan, 1995, p.4.

³⁹See Orr, Edey, and Kennedy (1995) and Orr (1995).

Next, we would like to explore the issue of the effects of monetary regime changes on the behaviour of real interest rates. Garcia and Perron (1994) argue that (for U.S. real interest rates) non-stationarity in real rates is related to occasional regime shifts. Bosner-Neal (1990), however, argues that shifts in monetary and exchange rate regimes are not related to changes in the behaviour of real interest rates. Since most of these arguments were based on U.S. data, we would like to examine similar issues in a Canadian context.

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Appendix 1

Serial Correlation Tests

A variety of tests can be performed to determine whether the residuals from the *ex post* real rate regressions follow an MA(2) process, which indirectly assess the adequacy of the specification of X_t .

(i) Ljung-Box Q Test

If the residuals from the *ex post* regressions follow an MA(2) process, then the residuals from an MA(2) model should not be serially correlated. We can test whether the residuals from an MA(2) model of the *ex post* residuals are not serially correlated by constructing the Q test-statistic. In this case, the null hypothesis of no serial correlation is tested against the alternative that serial correlation exists.

Table 8. Q Test Statistics

Test-Statistic	Q(3)=2.832	Q(6)=7.916	Q(12)=21.392	Q(24)=59.808*
Critical Value	6.635	13.277	23.209	40.298

The asterisk denotes significance at a level of 1%. Based on the first three test-statistics we are unable to reject the null hypothesis, however, the Q value computed at the twenty-fourth autocorrelation (Q(24)) rejects the null hypothesis.

(ii) F Tests

Breusch and Godfrey provide an alternative test for serial correlation which is less sensitive to the specification of the test-statistic. Instead of computing LM test statistics in the conventional manner from the residual regressions, we perform F -tests on the lagged residuals in the "artificial" regressions.⁴⁰ First we test for serial correlation of the MA(2) form on the residuals from the *ex post* real rate regressions and then we conduct tests of this form against other MA processes.

⁴⁰Davidson and MacKinnon, 1993, p.359 suggest that the F -tests are more appropriate for finite samples. Running an "artificial" regression consists of regressing the *ex post* residuals on the original explanatory variables plus lagged *ex post* residuals.

Table 9. *F* Test Statistics

Test Number	(1)	(2)	(3)	(4)
Null Hypothesis	No Serial Correlation	MA(2)	MA(2)	MA(2)
Alternative Hypothesis	MA(2)	MA(1)	MA(3)	MA(4)
Test Statistics	152.416* (4.61)	0.0244 (6.63)	1.6011 (3.78)	2.5837 (3.48)

The critical values (in parentheses) were chosen from the upper 1% points of the *F*-distribution. An asterisk denotes significance at 1%. In the first test the null hypothesis is easily rejected in favour of serial correlation in the MA(2) form. Tests (not shown here) of the null hypothesis in (1) against other MA forms of serial correlation, also rejected the null hypothesis. In order to determine the appropriate form of serial correlation, we tested the null hypothesis that the *ex post* residuals followed an MA(2) process against alternative hypotheses with higher and lower orders of serial correlation. In tests (2) through (4) we were unable to reject the null hypothesis that the *ex post* residuals followed an MA(2) process.

Test results from (i) and (ii) provide support for the MA(2) characterisation of the residuals from the *ex post* real interest rate regressions. This suggests that the specification of the X_t variables is adequate.

Appendix 2

Parameter Stability

Parameter estimates from the rolling ARMA regressions remained relatively stable across the sample period. The starting point of the estimation period was fixed at 1946:01 while the end-point advanced by one month after each inflation forecast was completed. The following graphs plot the coefficient estimates across the sample period. The value of the estimate at 1970:01 corresponds to the estimate from the regression over the period 1946:01 to 1970:01.

Figure 14. Constant Parameter

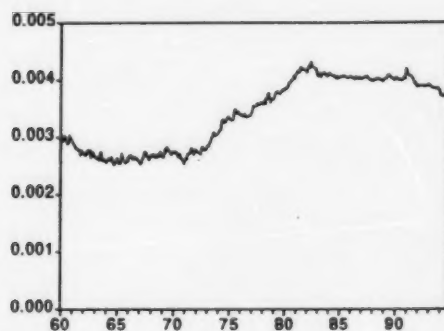


Figure 15. AR(1) Parameter

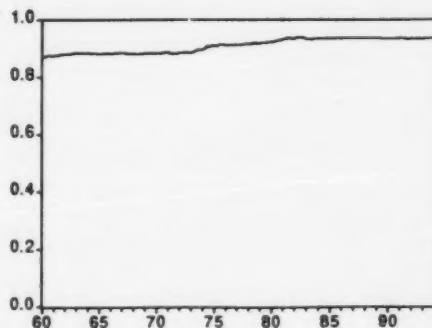


Figure 16. MA(1) Parameter

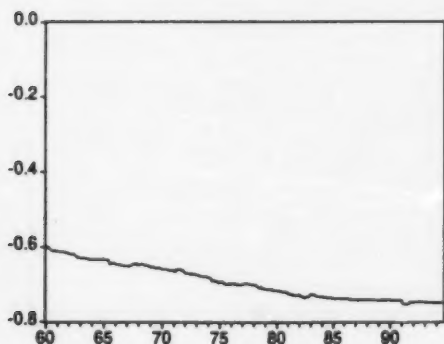
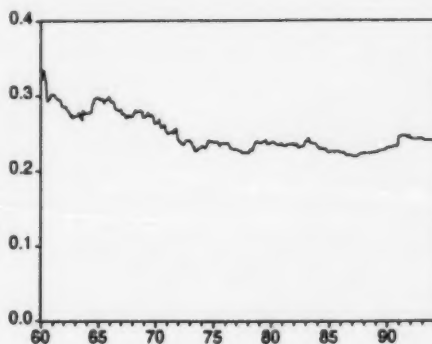


Figure 17. MA(12) Parameter



Appendix 3

Conventional Unit Root Tests

Table 10. Augmented Dickey-Fuller Unit Root Tests

Real Rate Series	ADF Unit Root Test-Statistics		
(monthly series)	ADF(F)	ADF(E)	DF
XARM	-2.1952	-2.6249	-2.6968
XATB	-2.1147	-3.6099* (0.018)	-3.7083* (0.242)
XAHP	-1.4449	-1.9853	-2.0481
(quarterly series)	ADF(F)	ADF(E)	DF
XARM	-2.0506	-2.3933	-2.2079
XATB	-1.8854	-2.4967	-2.8250
XAHP	-1.2983	-1.2381	-1.3597

Note: The letter in parentheses refers to the choice of lag length. (F) indicates a fixed lag length of 12 for the monthly series and 4 for the quarterly series. (E) indicates an endogenous choice of lag length, chosen according to Perron's (1990) method. * denotes significance at 5% using the MacKinnon critical values. Values in parentheses are marginal significance levels obtained from Monte Carlo simulations. In all the test regressions a constant was included.

Using the MacKinnon critical values, the hypothesis of non-stationarity cannot be rejected for all of the series (frequency and lag length choice) except XATB. For monthly figures, ADF(E) and DF are significant at 1% using the usual critical values, however, if we allow for the real rate to follow a mixed process, those critical values are not appropriate. Monte Carlo simulations were performed which generated a distribution of test-statistics where the underlying process for the real rate was mixed. From this distribution marginal significance levels were calculated. Based on the marginal significance levels we cannot reject the hypothesis of non-stationarity for DF (both quarterly and monthly series), although for ADF(E) we can reject the hypothesis at a 2% level of significance for XATB. Overall, the results support the hypothesis of non-stationarity.

Appendix 4

Parametric Calculations of Impulse Responses

Campbell and Mankiw (1987) estimate the impulse responses for U.S. GNP. Their approach is easily extended to estimate the impulse responses of the real interest rate. Suppose r_t can be expressed as the infinite MA process,

$$r_t = \Psi(L)\epsilon_t \quad (\text{A.1})$$

where $\Psi(L)$ is the impulse response function for r_t . The process above could also be expressed as the ARIMA model, where $\Phi(L)$ and $\Theta(L)$ are the usual AR and MA lag polynomials.

$$(1-L)\Phi(L)r_t = \Theta(L)\epsilon_t \quad (\text{A.2})$$

Substituting (A.1) into (A.2) yields

$$(1-L)\Phi(L)\Psi(L) = \Theta(L) \quad (\text{A.3})$$

To obtain the (lag weights ψ_i) impulse response function, equate the coefficients of L^i in the expansion of (A.3) for large i .

$$(1-L)(1-\phi_1L-\dots-\phi_pL^p)(1-\psi_1L-\psi_2L^2-\dots) = (1+\theta_1L+\dots+\theta_qL^q) \quad (\text{A.4})$$

The impulse response (for large k) ψ_k , is the long-term forecast of r_{t+k} given a 1% shock to r_t . If ψ_k is (greater than or) equal to one, forecasts of r_{t+k} increase by (at least) 1% in response to the shock, indicating that r_t is likely generated by a process such as (4.1). If ψ_k (for large k) is equal to zero, then long term forecasts of r_{t+k} are not affected by the shock to r_t and a stationary process is the appropriate characterisation.

It is apparent from this procedure that calculations of the impulse responses in this manner will be dependent on the specification of the parameters in the ARIMA representation (A.2). Campbell and Mankiw calculate the impulse response functions for various ARIMA(p,1,q) models of GNP where p and q take on the values {0,1,2,3}. Their results suggest that the impulse responses are between 1.3 and 1.9 at ($k=$) 80 quarters. However, for certain ARIMA models, the impulse responses are close to zero at the same horizon.

Appendix 5

Variance Ratio Estimates

Figure 4 presented the variance ratio estimates for the long-term real interest rate XARM. We provide variance ratio estimates for the short-term real rate series XATB and the medium term XAHP series in Figures 18 and 19 along with their confidence intervals.

Figure 18. Variance Ratio Estimates for XATB

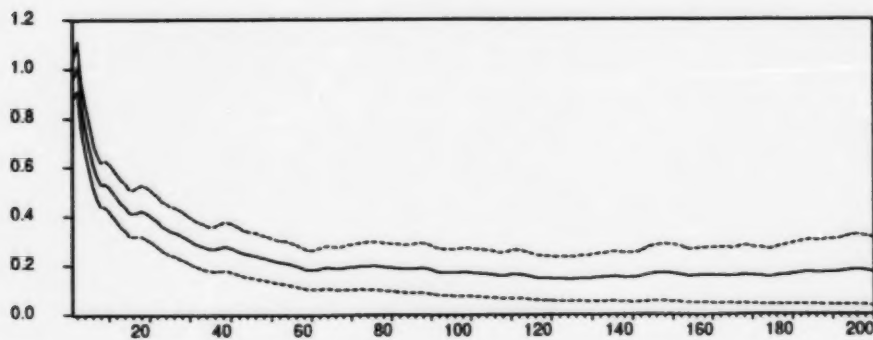
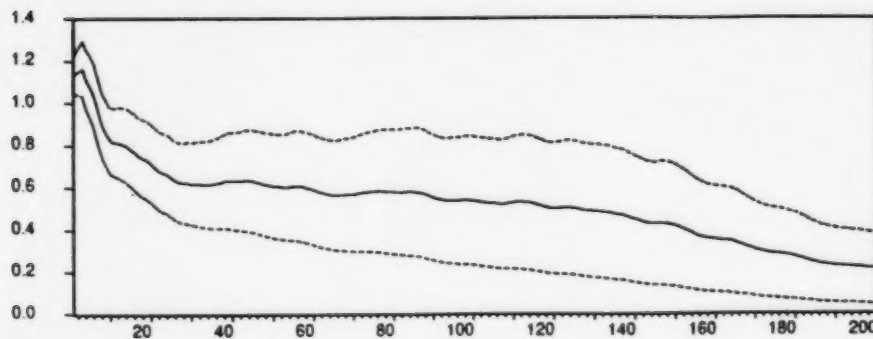


Figure 19. Variance Ratio Estimates for XAHP



Variance ratio calculations were also performed on the quarterly series for each rate. The point estimates did not change significantly, however, the confidence intervals were in some cases wider than those for the monthly series above.

Appendix 6

Impulse Response Estimates

Values of the impulse responses were also calculated for the various real interest rates. The variance of the shocks to the random walk component (used in the variance ratio calculations) is related to the impulse response function by,

$$\sigma_{\Delta r}^2 = \left(\sum_{j=0}^{\infty} \psi_j \right)^2 \sigma_{\epsilon}^2. \quad (\text{A.5})$$

The value of ψ_k at $k = 200$ (for monthly data and $k = 66$ for quarterly) represents the response of long-term forecasts of the real rate to a 1% shock.

Table 11. Impulse Response Estimates

Series	XARM	XATB	XAHP
$\hat{\psi}_{k=200}$ (monthly series)	0.40499 (0.1314)	0.41471 (0.1381)	0.46394 (0.1695)
$\hat{\psi}_{k=66}$ (quarterly series)	0.64317 (0.3078)	0.40533 (0.1321)	0.76015 (0.4560)

Note: asymptotic standard errors of each estimate are in parentheses.

Appendix 7

The Non-linear System of Equations

By normalising the variance of the disturbances to one and imposing the restriction that nominal shocks do not affect the real interest rate in the long-run, we are able to just identify the four elements in the A_0 matrix. This involves solving a non-linear system of equations:

$$\begin{aligned} \text{(i)} \quad & a_{11}^2 + a_{12}^2 = \text{var}(e^{\Delta r}) \\ \text{(ii)} \quad & a_{21}^2 + a_{22}^2 = \text{var}(e^{U-U^*}) \\ \text{(iii)} \quad & a_{11}a_{21} + a_{12}a_{22} = \text{cov}(e^{\Delta r}, e^{U-U^*}) \\ \text{(iv)} \quad & \left[\sum_{i=0}^{\infty} d_{11i} \right] a_{11} + \left[\sum_{i=0}^{\infty} d_{12i} \right] a_{21} = 0 \end{aligned}$$

where the e vectors represent the residuals and the d elements represent the impulse responses from the VAR regression.

Appendix 8

Stationarity of the Unemployment Gap

The assumption that the unemployment gap is stationary implies that nominal and real shocks will not have a permanent effect on the gap. Augmented Dickey-Fuller unit root tests provide support the hypothesis of stationarity. Tests were performed with the lag length chosen endogenously as well as with a fixed lag length of 4 (using quarterly data) in all cases without a constant term. Monte Carlo simulations were performed where the underlying process for the unemployment gap was mixed and a distribution of test-statistics were computed.

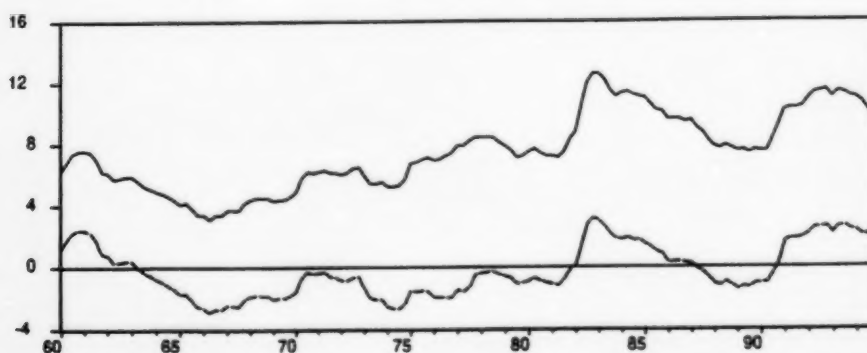
Table 12. ADF Unit Root Test Results on the Unemployment Gap

Lag Choice	Fixed Length	Endogenous Length
ADF test-statistic	-2.5810* (0.009)	-2.7133* (0.008)
Conclusions	- significant at 1%	- significant at 1%

Note: * denotes significance at 1% using MacKinnon critical values. Values in parentheses are marginal levels of significance obtained from Monte Carlo simulations. In the test regressions a constant was not included.

The following graph plots the unemployment rate in per cent (solid line) along with the unemployment gap (broken line). Estimates are based on quarterly data.

Figure 20. The Unemployment Rate and the Unemployment Gap, 1960-1994



Appendix 9

Impulse Responses and Variance Decompositions for Short and Medium Term Real Rates

The Blanchard-Quah approach was also applied to the short-term real interest rate series XATB and the medium term series XAHP. Since we are primarily interested in the behaviour of the real interest rate, we present only the impulse responses and variance decompositions for the real rate. The impulse responses and variance decompositions for the unemployment gap look similar to those presented above (see pages 24 and 25).

Real interest rate series XATB:

Impulse Responses

Figure 21. Real Rate Response to Nominal Disturbances

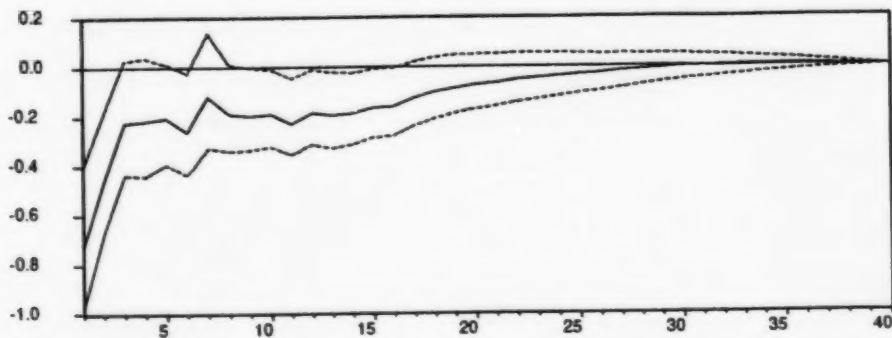
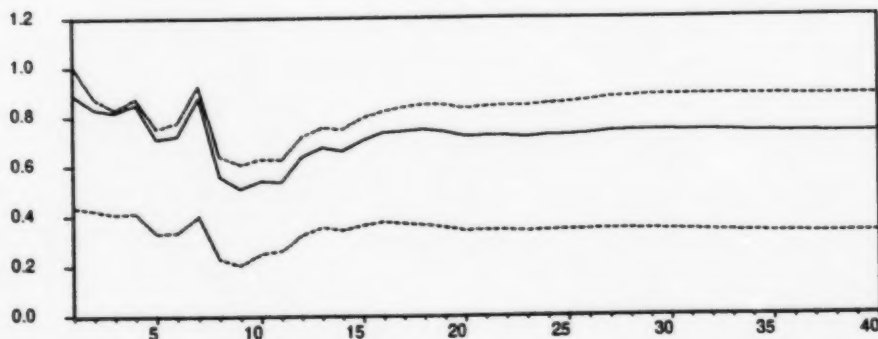


Figure 22. Real Rate Response to Real Disturbances



Variance Decompositions

Table 13. Percentage of Expected Squared Forecast Error Produced by Nominal and Real Shocks

Horizon	Real Interest Rate	
	Nominal	Real
1	40.0 (17.4, 79.4)	60.0 (20.1, 82.6)
2	33.2 (13.7, 72.9)	66.8 (27.1, 86.3)
3	26.8 (11.3, 65.2)	73.2 (34.8, 88.7)
4	22.5 (9.5, 59.7)	77.5 (40.3, 90.5)
8	16.7 (7.0, 50.1)	83.3 (49.4, 93.0)
12	15.7 (6.4, 47.8)	84.3 (52.2, 93.6)
40	6.0 (2.2, 19.5)	94.0 (97.8, 80.5)

Real interest rate series XAHP:

Impulse Responses

Figure 23. Real Rate Response to Nominal Disturbances

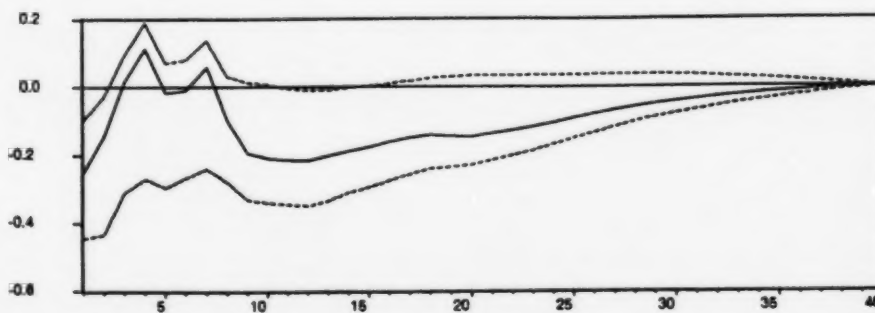
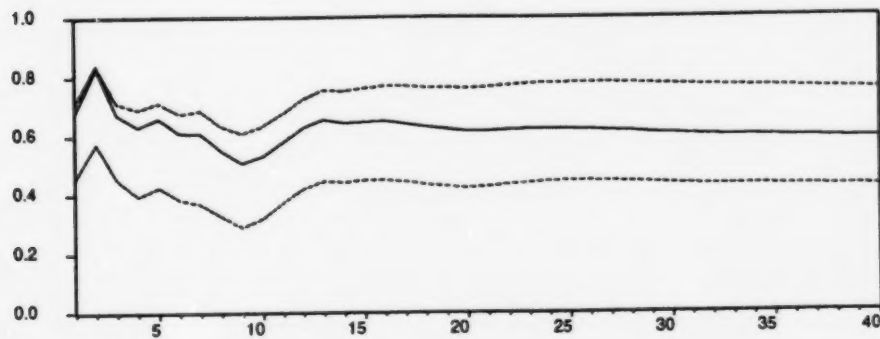


Figure 24. Real Rate Response to Real Disturbances



Variance Decompositions

Table 14. Percentage of Expected Squared Forecast Error Produced by Nominal and Real Shocks

Horizon	Real Interest Rate	
	Nominal	Real
1	11.8 (0.0, 46.5)	88.2 (53.5, 100.0)
2	6.7 (0.0, 39.3)	93.3 (60.7, 100.0)
3	4.9 (0.0, 35.2)	95.1 (64.8, 100.0)
4	4.6 (1.0, 32.8)	95.4 (67.2, 98.9)
8	3.1 (0.7, 30.2)	96.9 (69.8, 99.3)
12	5.6 (1.5, 31.0)	94.4 (69.0, 98.5)
40	3.7 (0.0, 16.0)	96.3 (84.0, 100.0)

The impulse responses in Figures 21 to 24 look similar to those in Figures 5 and 6. However, the long-run effect on the real rate in response to a real shock is approximately 10 to 15 basis points higher for the series XATB and XAHP in comparison to XARM. The variance decompositions for the series XATB were similar to those in Section IV. However, for the series XAHP, nominal shocks did not significantly contribute to the behaviour of the real interest rate at short and medium horizons.

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